**OPIM 5603: Statistics in Business Analytics**

**Assignment 3 - Spring 2023**

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**Problem 1 (35 Points) — Probability**

Suppose P(X) = 2/3 and P(Y) = 1/4

1. What is the maximum value of P (X ∩ Y)? => **0.25**

(b) What is the minimum value of P (X ∩ Y)? => **0**

(c) What is the maximum value of P (X ∩ Y′)? => **0.66**

(d) What is the minimum value of P (X Y)? => **0.66**

(e) What is the maximum value of P (X | Y)? => **1**

(f) What is the minimum value of P (X | Y)? => **0**

Note that Y’ indicates not Y.

1. P (X∩ Y) = 1/4 => 0.25

Diagram, venn diagram

Description automatically generated

If X has probability of 2/3 and B has probability of 1/4, then the max probability that can be attained can’t be greater than the values of (1/4 & 2/3) and maximum that can be possible is 1/4. The common sharing portion can’t be higher than the value of P(A) or P(B).

1. The minimum value of P (X ∩ Y) is 0, considering if they are mutually exclusive.
2. P (X∩ Y’) => 0.66

If X has probability of 2/3 and Y’ has probability of 3/4, then the max probability that can be attained can’t be greater than the values of (3/4 & 2/3) and maximum that can be possible is 2/3(i.e., 0.66). The common sharing portion can’t be higher than the value of P(A) or P(B).

1. Minimum of P (X U Y) = 0.66, to find the minimum, we need a max value for P(X∩ Y) i.e., 0.25

* P(A)+P(B)- P (X∩ Y) = 0.66
* 0.66+ 0.25 – 0.25 = 0.66

1. Maximum of P(X|Y) =1, that max value that can be attained is 1.
2. Minimum of P(X|Y) = 0, the min value that can hold is 0.

**Problem 2 (30 Points) — Bayes**

Suppose you are feeling symptoms that make you concerned that you may have a particular disease. The disease is only present in 2% of the population. You can take a test to determine whether or not you have the disease, but the test is inconclusive. If you have the disease the test will say you have the disease with probability 0.8. If you do not have the disease, the test will say you do not have the disease with probability 0.9. Conditional on you take the test and it says you have the disease, what is the probability that you do not have the disease?

D=> Disease T=> Test Positive

D’ => No Disease T=> Test Negative

P(D)= 0.02

P(D’) =0.98

P(T|D) =0.8 that implies P(T’|D) =0.2

P(T’|D’) =0.9 that implies P(T|D’) =0.1

Considering the hypothesis for 1000 (N) people the distribution is as follows:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Disease P(D) | No disease P(D’) | Total |
| Test+ P(T) | 16 P(T|D) \*P(D) | 98 P(T|D’) \*P(D’) | 114 P(T|D) + P(T|D’) |
| Test- P(T’) | 4 P(T’|D) \*P(D) | 882 P(T’|D’) \*P(D’) | 886 P(T’|D) + P(T’|D’) |
|  | 20 P(D) 1000\*0.2 | 980 P(D’) 1000\*0.8 | 1000 (N) |

The probability that you do not have the disease = P(D’|T) = P(D’∩T) /P(T)

* P(T/D’) \* P(D’) / P(T)
* P(T/D’) \*P(D’)/ [ P(T/D’)\*P(D’) + P(T|D) \*P(D)]
* 0.1 \* 0.98 / [ ( 0.1\*0.98) + (0.8\*0.02) ]
* 0.098/ 0.098+0.016
* 0.098/ 0.114
* 0.859

(Or)

P(D’|T’) = P(D’∩T) /P(T) => 98/114 = 0.859

**Problem 3 (35 Points) — Independence**

If a company has three potential investments and the investments are either successful or not. The success of an investment is independent of each other. Assume that each investment has probability 1/4 of being successful for the following sub questions (a)-(c).

(a) What is the probability that the second investment is successful?

(b) What is the probability that the third investment is successful, given that the three investments are either all successful or all not successful?

(c) What is the probability that the third investment is successful, given that two of the three investments are successful?

P (A and B) = P(A) \* P(B) for independent events

**a)** P(S2) = 1 /4 = 0.25 is ¼ (since for each investment probability of being successful is fixed)

**b)** P(S3) = P (3rd investment successful | all successful or all not successful)

=> P (3rd investment being successful) =1/4 ( independent event )

=> P (all successful or all not successful) => (1/4) ^3 + (3/4)^3 = 7/16

P (3rd investment successful | all successful or all not successful) =

=> P (all su) /P (all su or all not su)

=> (1/64)/ (7/16)

=> 1/28

**c)** P(2/3 investments are su) = P(n,s,s)+P(s,n,s)+P(s,s,n)

=> (3/64)^3 = 9/64

P( 3rd investment ∩ 2/3 investments are su) = P(s,n,s)+P(n,s,s)

=>3/32

P(3rd investment successful | 2/3 investments are su) (3/32 / 9/64) => 2/3

2/3

